Renewable Distributed Generations' Uncertainty Modelling: A Survey

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***Abstract*—Renewable energy distributed generation is reaching an unprecedented level of integration into power generation systems due to its numerous advantages. However, its increased penetration compounds the level of uncertainties being coped with in distribution systems. This aggravates the difficulty in making decisions in the context of large-scale penetration of renewable distributed generations, especially with the intermittent ones. Consequently, the analysis of uncertainty and modelling of the related system parameters is essential. This paper aims to provide a state-of-the-art review on uncertainty modelling approaches for distribution system studies and applications. This work focuses mainly on classifying and comparing the uncertainty modelling approaches and methodologies, presenting mathematical syntax of the methods, as well as the merits and demerits of the modelling methods. This study serves as the knowledge warehouse and selection tool for choosing the most suitable method for various applications.**

***Index Terms*—distributed generations, mathematical syntax, renewable energy resource, uncertainty modelling methods, uncertainty parameters**

I. Introduction

The planning of distributed generation (DG) problem in a distribution network system (DNS) involves many sources of uncertainty and variability, especially with the integration of intermittent renewable energy resource (IRES) [1]. These are due to randomness and variability these resources during the times of operations [2]. The variabilities and uncertainties caused by the intermittent hybrid DGs (solar PV and wind) power outputs, and demand, market and other power system uncertainties respectively, are usually considered during several operational states for modelling to cushion their impacts. The parameters that can be modelled in the planning and operation of renewable DGs to account for the uncertainties inherent in the distribution systems are discussed below [3], [4].

- Technical Uncertain Parameters: They are are either topological parameters or operational parameters. The topological parameters relate to network structures such as line outages, generation outage, instrumentation or devices failure, e.t.c. The operation uncertain parameters are those involved in the operating decisions such as uncertainty in generation values, and demand values in the distribution systems.
- Economic Uncertain Parameters: These parameters are uncertainty in the cost of production, fuel supply, business taxes, market prices that are analysed in microeconomics term. Furthermore, uncertain parameters such as economic growth, interest rates, inflation rates, regulation, deregulation, gross domestic product (GDP) and unemployment rates are analysed as macroeconomics indices.

These parameters contain a measure of uncertainties that should be appropriately addressed, especially in the renewable distributed generation power systems.

There have been numerous methodologies and approaches that have been used for modelling the aforementioned uncertain parameters, as will be discussed in Sections II-VII. All the uncertainty modelling methods are prioritised in power systems to quantify the input parameters' potentials and impacts, and predict their models' outputs. The major difference amongst these approaches is based on the type of function applied to describe the uncertainty of input parameters.

This paper seeks to present a brief state-of-the-art survey on uncertainty modelling techniques in distribution power system applications. It focuses mainly on classifying and comparing the uncertainty modelling approaches and methodologies, presenting mathematical syntax of the methods, as well as the merits and demerits of the modelling methods.

II. Probabilistic Approach

In 1955, Dantzing presented one of the earliest renowned works on probabilistic approach [5]. In his work, it was assumed that the probability distribution function (PDF) of input parameters variables are known before the output parameters variables can be obtained. Based on several methods introduced in the literature, the probabilistic approach is broadly classified into two major methods - numerical and analytical methods:

A. Numerical Probabilistic Methods

Monte Carlo Simulation (MCS) is one of the most common, reliable and accurate stochastic modelling methods. MCS is a system-size independent method that is applicable to non-linear, complex and or multi-uncertain variables systems [6]. MCS methods have been widely used in modelling power distribution system uncertainties [7]–[10]. The general mathematical syntax of MCS follows the iterative procedure in these steps:
Step 1: Let counter (K) for MCS be 1 i.e. $K = 1$
Step 2: Create a random sample for vector X by using PDF of individual element x_i of X.
Step 3: Determine y_k, by initializing $X = X_k$ while $y_k = f(X_k)$.
Step 4: Determine the expected value of y $(E(y) = \frac{\sum_i y_k}{K})$
Step 5: Determine the variance of y $(\sigma(y) = E(Y^2) - E^2(Y))$.
Step 6: Stopping criteria met? End; Else set K=K+1, then go to 2.
Step 7: End.

B. Analytical Probabilistic Method

Analytical methods normally produce arithmetic equations that are applied on PDFs of stochastic inputs to model the uncertain parameters. These methods are grouped into linearisation- and PDF approximation-based methods.

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1) Linearisation-Based Analytical (LBA) Methods: The LBA methods are dependent on linearisation of stochastic input variables. The methods under this umbrella are:

i) Convolution Method: Convolution method is a method being applied to analyse a system and obtain the PDFs of the system random input variables [3]. The syntax of a typical convolution method is as follows:
For n independent random variables X, Z is the uncertain parameter of the power system.
Let $X_1, X_2, \dots X_n$ be the random variables, and $fx_1(X_1), fx_2(X_2) \dots fx_n(X_n)$ be their PDFs.
Then, $Z = a_1X_1 + a_2X_2 + \dots + a_nX_n$; where $a_1, a_2, \dots, a_n$ are coefficient of Z.
Let $Y_i = a_iX_i$ be the output variable. Hence, the PDFs of Z is given as:

$$f_z(Z) = \frac{1}{a_1} fx_1 \left(\frac{Y_1}{a_1}\right) \circledast \dots \frac{1}{a_n} fx_n \left(\frac{Y_n}{a_n}\right) \quad (1)$$

The weakness with this method is the requirement for a large storage and high computational time when large or complex systems are involved.

ii) Cumulants Method: Cumulant method, unlike convolution, uses a simpler arithmetic process for determining the PDF of a linear combinations of many uncertain variables. This is done by utilizing moments and cumulants to extract features from probability distribution instead of complicated convolution computation [6]. The general syntax for cumulant method is given in these steps:
Step 1: Obtaining moment. Let γ^{th} moment of X be given as:

$$\alpha_\gamma = \int_{-\infty}^{+\infty} X^\gamma dF(X) \quad (2)$$

where X is a continuous random variable; γ, the power of the moment; $F(\gamma)$, the CDF of X. The central moments are the moments around the mean value, μ, of X, given as:

$$\beta_\gamma = E\left[(X-\mu)^\gamma\right] = \int_{-\infty}^{+\infty} (X-\mu)^\gamma dF(X) \quad (3)$$

The γ^{th} moment of X for a discrete random variable X, where the probability P_r exist for a matching element X_k of X, is given as:

$$\alpha_\gamma = \int_{r-1}^{\infty} P_r X_r^\gamma \quad (4)$$

Step 2: Obtaining cumulants: Instead of cumulative PDF, closed forms recursion can be applied on random variable to obtain cumulant k.

Step 3: Determining the PDF: After obtaining the moment and cumulant, the next is to determine the PDF. The approximation of the actual PDF is done by using many types of series expansion approaches where the coefficients of expansions are computed from the moments and cumulants of the distribution.

The limitation of the afore-mentioned linearisation-based analytical probabilistic methods is that the linearisation approximation obtained is not reliable because the error propagation cannot be well approximated using linear function.

2) PDF Approximation-Based Analytical (PABA) Methods: This group of analytical methods are dependent on PDF approximation. The benefit of using these methods is the ease of approximating a PDF over a non-linear transformation. The main principle of these methods lies on generating suitable representatives of input variables that can keep enough detail about the PDF of input variable. The methods under PDF approximation methods are:

i) Point Estimation Method (PEM): PEM refines the statistical information given by the few foremost central moment of a random variable on j points for the respective variable, called concentration. Point estimation algorithms (PEAs) are non-iterative, highly convergent and computationally efficient. The mathematical syntax for PEM is as follows:
Step 1: Assigning the initial values: Let $E(Y)^l = 0$; $E(Y^2)^l = 0$ and $l = 1$.
Step 2: Obtaining the location and probability of the two concentrations $X_{l,j}$ using (5) and (6) respectively as given in:

$$\varepsilon_{l,j} = \frac{\lambda_{l,3}}{2} - \sqrt{n + \left(\frac{\lambda_{l,3}}{2}\right)^2}; l = 1, \dots, n; j = 1, 2 \quad (5)$$

$$P_{l,j} = -\frac{\varepsilon_{l,3-j}}{2n\sqrt{n + \left(\frac{\lambda_{l,3}}{2}\right)^2}}; \; l = 1, \dots, n; \; j = 1, 2 \quad (6)$$

Step 3: Determining the concentration points $X_{l,j}$

$$X_{l,j} = \mu_{X\,l} + \varepsilon_{l,2}.\gamma_{X\,l}; \; l = 1, \dots, n; \; j = 1, 2 \quad (7)$$

where $\mu_{X\,l}$ and $\gamma_{X\,l}$ are the mean and variance of uncertain variable l, respectively.
Step 4: Determine the output variable, f, in respect of $X_{l,j}$

$$X = [x_1, x_2, \dots, X_{l,j}, \dots, x_n]; \; j = 1, 2 \quad (8)$$

Step 5: Determine $E(Y)^{l+1}$, $E(Y^2)^{l+1}$ for all random variables using:

$$E(Y)^{l+1} = E(Y)^l + \sum_{j=1}^{2} P_{l,j} f(X) \quad (9)$$

$$E(Y^2)^{l+1} = E(Y^2)^l + \sum_{j=1}^{2} P_{l,j} f^2(X) \quad (10)$$

Step 6: If $l < n$ $l = l + 1$, then go to Step 2; Else continue.
Step 7: Determine μ and σ of Y as:

$$\mu_Y = E(Y); \quad \sigma_Y = \sqrt{E(Y^2) - \mu^2(Y)} \quad (11)$$

Point estimation method has been applied variously in power distributed generations uncertainty applications [11]–[13]. However, point estimation methods only extract the means and standard deviations of the uncertain variables without giving any information on the shape of the PDFs of the output variables.
So far, all the afore-discussed analytical methods are very weak in finding correlation among the uncertain variables of a complex system. Moreover, correlation among uncertain variables is very consequential in the present day power systems studies.

ii) Unscented Transformation Method (UTM): UTM was introduced to overcome the shortcomings of the conventional probabilistic methods especially those that are based on linearisation process. UTM is a robust method for evaluating stochastic problems with or without corresponded uncertain variables. UTM is very dependable for determining the statistics of output uncertain variables that requires non-linear transformations. This method can be applied to get the mean and covariance of the output variable of two or more correlated random variables [14]. Let X be multi-dimensional random variables whose mean and covariance are $\bar{X}$ and P_{Xl} respectively. Let another uncertain variable Y relates to X via a non-linear function $Y = f(X)$. Using UTM, the mean, $\bar{Y}$ and covariance, P_{YY} of output variable, Y, can be obtained with these simple steps:

Step 1: Obtaining sigma points (2n+1 samples) of input variables using (12) - (14)

$$X^{\circ} = m \tag{12}$$

$$X^{l} = m + \left(\sqrt{\frac{n}{1-W^{\circ}}P_{Xl}}\right)_{l}; l = 1, 2, \ldots, n \tag{13}$$

$$X^{l+n} = m - \left(\sqrt{\frac{n}{1-W^{\circ}}P_{Xl}}\right)_{l}; l = 1, 2, \ldots, n \tag{14}$$

Step 2: Determine the weight of individual sample: (i) $W^{\circ} = W^{\circ}$, (ii) $W^{l} = \frac{1-W^{\circ}}{2n}; l = 1, 2, \ldots, n$ and (iii) $W^{l+1} = \frac{1-W^{\circ}}{2n}; l+n = n+1, \ldots, 2n$. where W° is the weight assigned to the point $\bar{X} = m$, called the zeroth point.
Step 3: Supply respective sample point to the non-linear function to obtain some transformed sample points from:

$$y^{l} = f(x^{l}) \tag{15}$$

Step 4: Determine $\bar{Y}$ and P_{YY} of output variable Y from (16).

$$\bar{Y} = \sum_{l=0}^{2n} W^{l}Y^{l}; \quad P_{YY} = \sum_{l=0}^{2n} W^{l}(Y^{l}-\bar{Y})(Y^{l}-\bar{Y})^{T} \tag{16}$$

UTM has been variously implemented to assess power system uncertainty parameters [15], [16].
However, UTM is deficient in being applicable to only problems whose input variables are expressed with their PDFs.

iii) Scenario-Based Decision Making (SBDM) Method: The uncertain variable normally have countless realisations but it is not possible to put all these realisations into consideration at the same time. Alternatively, the realisation space is split into a number of sections called scenarios that have a definite weight or probability. It is achieved by generating a number of scenarios with the PDF of an individual uncertain parameter, say X_s. The output variable, y, expected value, E(y) can be obtained from:

$$E(y) = \sum_{s} \Pi_{s} \times f(X_{s}) \tag{17}$$

where Π_s is the probability of s^{th} scenario, $\sum_s \Pi_s = 1$.
The scenario-based decision making method has computational efficiency and simplicity of implementation as some of its main strengths. SBDM finds many applications in power system uncertainty modellings [17]–[19]. However, the drawback of this method is its inability to give more statistical analysis other than only the expected values of the uncertain output variable.

However, the choice of suitable PDF for modelling uncertain variables is tasking, more so with imprecise or inadequate data [20]. In this sense, the application of possibility theory may serves as a viable alternative.

III. Possibilistic Approach

The principle of possibilistic uncertainty modelling is based on technique of fuzzy set theory that was introduced in 1965 by Zadeh where membership functions (MFs) are used to describe the input uncertain parameters [21]. Different kinds of MFs can be employed to model the membership degree of a possibilistic uncertain parameters. There are two sequence procedural techniques for modelling with possibilistic method [22].

A. α-cut technique

The uncertain output variable Y of a model, U, with uncertain input variables X' is normally described in power systems with a multi-variable function $Y = f(X_1', X_2', ...X_n')$. Once the uncertain input variable X' possibility distributions are given, the distribution of output variable Y can be determined using α-cut technique.
Given a fuzzy set A of U, the crisp set A^{α} consists of every individual of U that has membership value A as in:

$$A^{\alpha} = x \in U | \Pi_{X'}(X) \geq \alpha; \ 0 \leq \alpha \leq 1 \tag{18}$$

$$A^{\alpha} = [\underline{A}^{\alpha}, \bar{A}^{\alpha}] \tag{19}$$

where U is the universe of discourse of X', indicating ranges of X' possible values. $\underline{A}^{\alpha}$ and $\bar{A}^{\alpha}$, the lower and upper bounds of A^{α} respectively. Having determine the α-cut X^{α} of uncertain input variable X' with (18), the α-cut Y^{α} of uncertain output variable Y can be obtained from:

$$Y^{\alpha} = [\underline{Y}^{\alpha}, \bar{Y}^{\alpha}] \tag{20}$$

$$\underline{Y}^{\alpha} = (\min_{\bar{X}^{\alpha}} f(X_i^{\alpha})); \quad \bar{Y}^{\alpha} = (\max_{\bar{X}^{\alpha}} f(X_i^{\alpha})) \tag{21}$$

where X_i^{α} represents the α-cut of the i_{th} uncertain input variable.

B. Defuzzification

The defuzzification is a process of converting a fuzzy number into a crisp number [22]. The process is achieved using such techniques as centroid, maximum defuzzification and weighted average defuzzification techniques. Employing centroid technique, the defuzzified value of a fuzzy parameter, X' can be determined from:

$$X^{*} = \frac{\int \prod_{\bar{X}}(X) X dx}{\int \prod_{\bar{X}}(X) dx} \tag{22}$$

The possibilistic methods are widely applied in modelling of distributed generation systems uncertainties [23].

However, performance evaluation of uncertain parameters is more tasking than the deterministic ones especially when some variables have incomplete data. These issues become more pronounced when some variables have incomplete data. Meaning, such uncertain variables have some probabilistic and some possibilistic variables. Some hybrid methods are therefore developed to handle such cases.

IV. Hybrid Possibilistic-Probabilistic Approach

This approach is useful to model the uncertain systems that exhibit both possibilistic and probabilistic parameters simultaneously [14]. The methods under this hybrid approach structure possibilistic and probabilistic algorithms into two loops, the inner and outer loops respectively. These methods are discussed as follows:

A. Possibilistic-Monte Carlo Method

Under this method, MCS is the outer loop while α-cut is the inner loop. The statistical data of the output parameters MF such as expected value or PDF is obtained from the input parameters. The method is described with these steps:
Step 1: Generating a value with its PDF and Z_i^{e} for each $z_i \in Z$
Step 2: Calculating $\underline{Y}^{\alpha}$ and $\bar{Y}^{\alpha}$ by:

$$\underline{Y}^{\alpha} = (\min f(Z^{e}, X^{\alpha})); \quad \bar{Y}^{\alpha} = (\max f(Z^{e}, X^{\alpha})) \tag{23}$$

$$X^{\alpha} = (\underline{X}^{\alpha}, \bar{X}^{\alpha}) \tag{24}$$

The above procedure is iterated for obtaining the statistical data of the output parameters' MF such as expected value or PDF. This method has a shortcoming of being computational time intensive.

B. Possibilistic-Scenario-Based Method

Under this method, scenario-based algorithm is the outer loop while α-cut is the inner loop. This method is advantaged in being less computational complex and simpler to implement than the one presented in IV-A. The general procedure for describing this method are:
Step 1: Generating scenario set that describes the behaviour of Z_i, ω_j.
Step 2: Reducing the initial scenario set to a smaller one, ω_j.
Step 3: Calculating $\underline{X}^\alpha$, $\bar{X}^\alpha$:

$$\underline{Y}^\alpha = (\min \sum_{s\in\omega_s} \Pi_s.f(Z_s, X^\alpha)); \quad \bar{Y}^\alpha = (\max \sum_{s\in\omega_s} \Pi_s.f(Z_s, X^\alpha))$$

$$X^\alpha = (\underline{X}^\alpha, \bar{X}^\alpha) \tag{25}$$

Step 4: Defuzzifying Y.
However, possibilistic-scenario-based method only calculates output variable's means and its accuracy is very low.

C. Hybridised UTM-Fuzzy Set Theory Method

Under this method, fuzzy α-cut is the outer loop, for performing uncertainty analysis of possibilistic variables while UTM is the inner loop, for performing the uncertainty assessment of probabilistic variables [6].

In [24]–[27], hybrid possibilistic-probabilistic methods are differently proposed as tools for assessing the impact of uncertainties of electric loads and renewable and conventional DGs operations on technical performance of a distribution network.

V. Information Gap Decision Theory Method

Information gap decision theory (IGDT) was introduced in 1980 by Yakov Ben-Haim as a technique to model uncertain parameters that could not be described with either PDF or MF because of insufficient historical information [28]. It, however, measures the difference between the uncertainty of input parameters and their estimations. The syntax of a typical IGDT optimisation function is given as:

$$y = \min_d f(X, d) \tag{26}$$

$$G_i(X, d) = 0, i \in \Psi_{eq}; \quad H_j(X, d) \le 0, j \in \Psi_{ineq} \tag{27}$$

where X, the vector of uncertain parameter; d, the set of decision variables; H and G, the inequality and equality constraints respectively; and $f(X, d)$, defines the relationship between the input uncertain parameter X and decision variable d.

In IGDT, the uncertainty of parameters is normally described with envelop bound model [29]. The mathematical expression for the uncertainty set is given as:

$$\bar{X} = U(\sigma\alpha, \bar{X}); \quad U(\sigma\alpha, \bar{X}) = |\frac{X - \bar{X}}{\bar{X}}| \le \sigma\alpha \tag{28}$$

where $\sigma(\alpha)$, the largest allowable deviation level of uncertain input parameter actual value from its forecasted value; $\bar{X}$, the forecasted value of X; and $U(\sigma\alpha, \bar{X})$, the uncertain set of all values of X that deviate from $\bar{X}$ by not more than $\sigma\bar{X}$. $\sigma\bar{X}$ is also known as the radius of uncertainty which is usually unknown (σX) for the decision maker.

IGDT method has been variously applied in power system studies [30], [31].

VI. Robust Optimisation (RO) Method

Robust optimisation was introduced in 1973 by Soyster [32]. This method is developed to solve optimisation problems that are affected by uncertainty especially those that lack full information on the nature of uncertainty [33]. This is done by using uncertainty groups to describe the uncertainty associated with an input parameter. The mathematical description of RO is stated as:

Let a function $Z = f(X, Y)$ be linear in X and non-linear in Y. The values of X are uncertain while the values of Y are given. Robust optimisation thus uses the assumption that no specified PDF is available to describe the uncertain parameter X. Parameter X uncertainty is described with an uncertainty set $X \in U(X)$. $U(X)$ is the set from where parameter X takes values. Maximisation of Z=f(X,Y) is formulated as follows:

$$\max_Y Z = f(X, Y); \quad \forall X \in U(X) \tag{29}$$

The robust version of (29) is presented as:

$$\max_Y Z; \quad Z \le f(X, Y) \tag{30}$$

subject to:
(i) $\sum_j W_j \le \Gamma$; (ii) $0 \le W_j \le 1$ and

$$(iii) \quad f(X, Y) = A(Y) * \bar{X} + g(Y) - \max_{W_j} \sum_j a_j(Y) * \hat{X} * W_j$$

where $\bar{X}$, the forcasted value; $\hat{X}$, the maximum allowable deviation of X from $\bar{X}$. Robust optimisation has been widely applied to solve power system uncertainties during the integrations of renewable DGs [34]–[36].

VII. Interval Analysis (IA) Method

Interval analysis method was introduced in 1966 by Moore, based on the assumption that the uncertain input parameter is the extracted range of values from a known interval [37]. IA finds the upper and lower bounds for an uncertain input parameter. The method is similar to the probabilistic methods with a uniform PDF.
Let multivariate function $f = (x_1, \ldots, x_n)$
Subject to: $lb_j \le x_j \le ub_j$. lb_j and ub_j are the lower and upper limits of uncertain parameter x_j.

$$\begin{aligned} f &= \int_a^d A_1 \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \\ &= \frac{1}{\sigma\sqrt{2\pi}} \left[\int_a^b \frac{x-a}{b-a} e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_b^c e^{\frac{-(x-\mu)^2}{2\sigma^2}} \right. \\ & \quad \left. + \int_c^d \frac{x-d}{c-d} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \right] \end{aligned} \tag{31}$$

$$G(f) = \mu_2(f) \tag{32}$$

Interval analysis methods have found applications in modelling of uncertainties in renewable power injected distribution systems [38], [39].

VIII. Conclusion

This paper has provided a state-of-the-art review on uncertainty modelling methods being applied in handling distribution system uncertainties especially when renewable energy distributed generations are incorporated into the network. The methods have been classified into various groups that includes: probabilistic, possibilistic, hybrid possibilistic-probabilistic, information gap decision theory, robust optimisation and interval analysis. It is expected that this work will serve as a point of reference to researchers and decision makers working on integrating large-scale renewable distributed generations.

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